Differential Equations

Question1

The substitution required to reduce the differential equation $t^2 dx+\left(x^2-tx+t^2\right) dt=0$ to a differential equation which can be solved by variables separable method is

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Options:

A.

$t=V_x$

B.

$ax+bt=Z$

C.

$V=tx^2$

D.

$x=tV^2$

Answer: A

Solution:

Given, $t^2 dx+\left(x^2-tx+t^2\right) dt=0$

$\Rightarrow \quad \frac{dt}{dx}=-\frac{t^2}{x^2-tx+t^2}$

On putting $t=Vx$

$$\Rightarrow \quad \frac{dt}{dx} = V + x\frac{dV}{dx}$$

$$\therefore \quad \left(V + x\frac{dV}{dx}\right) = -\frac{V^2x^2}{x^2 - Vx^2 + V^2x^2}$$

$$\Rightarrow \quad V + x\frac{dV}{dx} = -\frac{V^2}{1 - V + V^2}$$

$$\Rightarrow \quad x\frac{dV}{dx} = -\frac{V^2}{1 - V + V^2} - V$$

$$\Rightarrow \quad x\frac{dV}{dx} = \frac{-V^2 - V + V^2 - V^3}{1 - V + V^2}$$

$$\Rightarrow \quad x\frac{dV}{dx} = -\frac{V + V^3}{1 - V + V^2}$$

$$\Rightarrow \quad \frac{1 - V + V^2}{V + V^3}dV = -\frac{dx}{x}$$

Which can be solve by variables separable method.

Question2

The equation which represents the system of parabolas whose axis is parallel to Y-axis satisfies the differential equation.

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Options:

A.

$\frac{d^3y}{dx^3} = 0$

B.

$\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} = x + y$

C.

$\frac{d^2y}{dx^2} + xy = 4ax$

D.

$\frac{dy}{dx} + xy = x^2$

Answer: A

Solution:

Equation of parabola with its axis parallel to the Y-axis.

$$y=ax^2+bx+c, a\neq 0$$
$$\Rightarrow \frac{dy}{dx}=2ax+b\Rightarrow \frac{d^2y}{dx^2}=2a$$
$$\Rightarrow \frac{d^3y}{dx^3}=0$$

Question3

If $\cos x\frac{dy}{dx}=y\sin x-1, x\neq(2n+1)\frac{\pi}{2}, n\in Z$ is the differential equation corresponding to the curve $y=f(x)$ and $f(0)=1$, then $f(x)$

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Options:

A.

$(1-x)\sec x$

B.

$(1-x)\cos x$

C.

$x+\cos x$

D.

$x+\sec x$

Answer: A

Solution:

Given, $\cos x\frac{dy}{dx}=y\sin x-1,$

$$x\neq(2n+1)\frac{\pi}{2}, n\in Z$$
$$\Rightarrow \frac{dy}{dx}=\frac{y\sin x}{\cos x}-\frac{1}{\cos x}$$

$$= y\tan x - \sec x$$

$$\Rightarrow \quad \frac{dy}{dx} - y\tan x = -\sec x$$

This is in the form $\frac{dy}{dx} + P(x)y = Q(x)$,

where, $P(x) = -\tan x$ and $Q(x) = -\sec x$

$$\text{IF} = e^{\int P(x)dx} = e^{\int -\tan x dx}$$
$$= e^{\ln|\cos x|} = |\cos x|$$

So, the general solution is

$$y \cdot \text{ IF } = \int A(x) \cdot \text{ IF } dx + C$$

$$y \cdot \cos x = \int -\sec x \cdot \cos x dx + C$$

$$= -\int dx + C = -x + C$$

$$= y = \frac{-x + C}{\cos x}$$

Given, $f(0) = 1$

$\Rightarrow \quad y = 1$ and $x = 0$

So, $1 = \frac{-0+C}{\cos(0)}$

$\Rightarrow 1 = C$

So, $y = \frac{-x+1}{\cos x} = (1 - x)\sec x$

Question4

The general solution of the differential equation $2dx + dy = (6xy + 4x - 3y)dx$ is

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Options:

A.

$2\log|2x - 1| = 3y^2 + 4y + C$

B.

$\log|3y+2| = 3x^2 - 3x + C$

C.

$\log|3y+2| = x^2 - x + C$

D.

$\log|2x-1| = 3y^2 - 4y + C$

Answer: B

Solution:

Given, differential equation

$$\begin{aligned}
&2dx + dy = (6xy + 4x - 3y)dx \\
\Rightarrow\quad & dy = (6xy + 4x - 3y - 2)dx \\
\Rightarrow\quad & \frac{dy}{dx} = 6xy + 4x - 3y - 2 \\
\Rightarrow\quad & \frac{dy}{dx} = 6xy + 4x - 2 - 3y \\
\Rightarrow\quad & 2x(3y+2) - 1(3y+2) \\
\Rightarrow\quad & (3y+2)(2x-1) \\
\Rightarrow\quad & \frac{dy}{3y+2} = dx(2x-1)
\end{aligned}$$

Integrating to both sides w.r.t x, we get

$$\begin{aligned}
&\int \frac{dy}{3y+2} = \int (2x-1)dx \\
&\Rightarrow \frac{1}{3}\log|3y+2| = \frac{2x^2}{2} - x + C_1 \\
&\Rightarrow \frac{1}{3}\log|3y+2| = x^2 - x + C_1 \\
&\Rightarrow \quad \log|3y+2| = 3x^2 - 3x + 3C_1 \\
&\quad = 3x^2 - 3x + C
\end{aligned}$$

(where $C = 3C_1$)

So, $\log|3y+2| = 3x^2 - 3x + C$

Question5

If the equation of the curve which passes through the point $(1,1)$ satisfies the differential equation $\frac{dy}{dx}=\frac{2x-5y+3}{5x+2y-3}$, then the equation of that curve is

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Options:

A. $x^2+5xy-y^2+3x-3y-5=0$

B. $x^2+5xy-y^2+3x+3y-11=0$

C. $x^2-5xy-y^2-3x-3y+11=0$

D. $x^2-5xy-y^2+3x+3y-1=0$

Answer: D

Solution:

$\frac{dy}{dx}=\frac{2x-5y+3}{5x+2y-3}$

Put, $x=X+h$

and $y=Y+K$

$\frac{dy}{dx}=\frac{dY}{dX}=\frac{2(X+h)-5(Y+K)+3}{5(X+h)+2(Y+K)-3}$

$\frac{dY}{dX}=\frac{2X-5Y+2h-5K+3}{5X+2Y+5h+2K-3}$

It is Homogeneous differential equation if

$2h-5K+3=0$

$5h+2K-3=0$

$$\frac{h}{15-6}=\frac{-K}{-6-15}=\frac{1}{4+25}$$

$$\frac{h}{9}=\frac{K}{21}=\frac{1}{29}$$

$$\Rightarrow \quad h=\frac{9}{29} \text{ and } K=\frac{21}{29}$$

$$\frac{dY}{dX}=\frac{2X-5Y}{5X+2Y}$$

Put $Y=vX$

$$v+X\frac{dv}{dX}=\frac{2-5v}{5+2v}$$

$$\frac{dY}{dX}=v+X\frac{dv}{dX}$$

$$X\frac{dv}{dX}=\frac{2-5v}{5+2v}-v=\frac{2-5v-5v-2v^2}{5+2v}$$

$$X\frac{dv}{dX}=\frac{2-10v-2v^2}{2v+5}=-2\frac{(v^2+5v-1)}{2v+5}$$

$$\int\frac{2v+5}{v^2+5v-1}dv=\int-\frac{2dx}{x}$$

$$\ln\left(v^2+5v-1\right)=-2\ln X+\ln C$$

$$\ln\left(v^2+5v-1\right)+\ln X^2=\ln C$$

$$x^2\left(v^2+5v-1\right)=C$$

$$X^2\frac{(Y^2+5XY-X^2)}{X^2}=C$$

$$Y^2+5XY-X^2=C$$

$$\left(y-\frac{21}{29}\right)^2+5\left(x-\frac{9}{29}\right)$$

$$\left(y-\frac{21}{29}\right)-\left(x-\frac{9}{29}\right)^2=C$$

Since, curve passes through point $(1,1)$

$$\left(1-\frac{21}{29}\right)^2+5\left(1-\frac{9}{29}\right)$$

$$\left(1-\frac{21}{29}\right)-\left(1-\frac{9}{29}\right)^2=C$$

$$\left(\frac{8}{29}\right)^2+5\left(\frac{20}{29}\right)\left(\frac{8}{29}\right)-\left(\frac{20}{29}\right)^2=C$$

$$\frac{1}{29^2}(64+800-400)=C$$

$$C=\frac{464}{29^2}=\frac{16}{29}$$

Equation of curve is

$$\left(y-\frac{21}{29}\right)^2+5\left(x-\frac{9}{29}\right)$$

$$\left(y-\frac{21}{29}\right)-\left(x-\frac{9}{29}\right)^2=\frac{464}{29^2}$$

$$y^2+\frac{441}{29^2}-\frac{42}{29}y+5\left[xy-\frac{9}{29}y-\frac{21}{29}x+\frac{189}{29^2}\right]-\left[x^2+\frac{81}{29^2}-\frac{18x}{29}\right]=\frac{464}{29^2}$$

$\Rightarrow y^2 - x^2 + 5xy + x$

$$\left(-\frac{105}{29} + \frac{18}{29}\right) + y\left(-\frac{42}{29} - \frac{45}{29}\right) + \frac{441 + 945 - 81}{29^2} = \frac{464}{29^2}$$

$\Rightarrow \quad y^2 - x^2 + 5xy - \frac{87}{29}x - \frac{87}{29}y + \frac{1305}{29^2} = \frac{464}{29^2}$

$\Rightarrow y^2 - x^2 + 5xy - \frac{87}{29}x - \frac{87}{29}y + \frac{841}{29^2} = 0$

$\Rightarrow \quad y^2 - x^2 + 5xy - \frac{87}{29}x - \frac{87}{29}y + 1 = 0$

$\Rightarrow x^2 - y^2 - 5xy + \frac{87}{29}x + \frac{87}{29}y - 1 = 0$

$x^2 - 5xy - y^2 + 3x + 3y - 1 = 0$

Question6

The general solution of the differential equation $\left(6x^2 - 2xy - 18x + 3y\right)dx - \left(x^2 - 3x\right)dy = 0$ is

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Options:

A. $2x^3 - x^2y - 9x^2 + 3xy + C = 0$

B. $4x^3 - 2x^2y - 6x^2 + 6xy + C = 0$

C. $2x^2 - 4xy - y^2 - x + 3y + C = 0$

D. $3x^2 + 5xy - 2y^2 - 4x - 2y + C = 0$

Answer: A

Solution:

$\left(6x^2 - 2xy - 18x + 3y\right)dx - \left(x^2 - 3x\right)dy = 0$

$M(x, y) = 6x^2 - 2xy - 18x + 3y$

On differentiate M with respect to y, whe

$$\frac{\partial M}{\partial y} = \frac{d}{dy}\left[6x^2 - 2xy - 18x + 3y\right]$$

$$= -2x + 3$$

$$N(x, y) = -\left(x^2 - 3x\right)$$

On differentiate N with respect to x, we get

$\frac{\partial N}{\partial x} = \frac{d}{dx}\left[-\left(x^2 - 3x\right)\right] = -(2x - 3)$

So, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

The two sites have been shown to be equivalent the equation is an identity any the differential equation is event. Now, let $\frac{\partial f}{\partial n} = M(x, y)$, then integrate with respect to xn, heat y as constant as follows

$\int \frac{\partial f}{\partial x} = \int \left(6x^2 - 2xy - 18x + 3y\right)\partial x$

$f = 2x^3 - \frac{yx^2}{2} - 9x^2 + 3xy + H(y)$

To solve for $H(g)$, let $\frac{\partial f}{\partial y} = N(x, y)$

Then, differentiate of with respect to y, heat x as constant as follows.

$$\frac{\partial f}{\partial y} = 0 - x^2 - 0 + 3x + \frac{\partial H(y)}{\partial y}$$

$$= -x^2 + 3x + \frac{\partial H(y)}{\partial y}$$

On using Eq. (ii), we get

$$-x^2 + 3x + \frac{\partial H(y)}{\partial y} = -\left(x^2 - 3x\right)$$

$$-x^2 + 3x + \frac{\partial H(y)}{\partial y} = -x^2 + 3x$$

$$\frac{\partial H(y)}{\partial y} = 0$$

Now, integrate the above term as $\int \partial H(y) = 0$

On substitute $H(y) = 0$ in the Eq. (i), we get

$\therefore$ We can equate of to C, we get $2x^3 - x^2y - 9x^2 + 3xy = C_1$

Hence, the general solution of the given differential equation is

$2x^3 - x^2y - 9x^2 + 3xy + c = 0$

Question7

The order and degree of the differential equation

$\frac{dy}{dx} = \left(\frac{d^2y}{dx^2} + 2\right)^{\frac{1}{2}} + \frac{d^2y}{dx} + 5$ are respectively

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Options:

A. 2,1

B. 2, 4

C. 2,2

D. 2,3

Answer: C

Solution:

To determine the order and degree of the differential equation, let's start with the given expression:

$$\frac{dy}{dx} = \left(\frac{d^2y}{dx^2} + 2\right)^{\frac{1}{2}} + \frac{d^2y}{dx^2} + 5$$

Rearrange the terms to isolate the square root:

$$\left[\frac{dy}{dx} - \frac{d^2y}{dx^2} - 5\right] = \left(\frac{d^2y}{dx^2} + 2\right)^{\frac{1}{2}}$$

Square both sides to eliminate the square root:

$$\left[\frac{dy}{dx} - \frac{d^2y}{dx^2} - 5\right]^2 = \frac{d^2y}{dx^2} + 2$$

Now, we have a polynomial in derivatives. The highest order derivative in this equation is $\frac{d^2y}{dx^2}$, indicating that the order of the differential equation is 2. Since the equation is a polynomial in terms of its highest derivative with the degree being that of the highest exponent applied to the derivative, and here the highest power is 2 (after squaring), the degree of the differential equation is also 2.

Therefore, the order and the degree of the differential equation are both 2.

Question8

If $y = \sin x + A\cos x$ is general solution of $\frac{dx}{dy} + f(x)y = \sec x$, then an integrating factor of the differential equation is

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Options:

A. $\sec x$

B. $\tan x$

C. $\cos x$

D. $\sin x$

Answer: A

Solution:

Given the equation $y = \sin x + A\cos x$, differentiate both sides with respect to x:

Substitute $A = \frac{y - \sin x}{\cos x}$ into the equation.

Differentiate:

$\frac{dy}{dx} = \cos x - \left(\frac{y - \sin x}{\cos x}\right)\sin x$

Simplify:

$\frac{dy}{dx} = \cos x - y\tan x + \frac{\sin^2 x}{\cos x}$

Further simplification leads to:

$\frac{dy}{dx} + y\tan x = \frac{\cos^2 x + \sin^2 x}{\cos x}$

Since $\cos^2 x + \sin^2 x = 1$, rewrite it as:

$\frac{dy}{dx} + y\tan x = \sec x$

Thus, the integrating factor is:

$e^{\int \tan x\, dx} = e^{\log(\sec x)} = \sec x$

So, the integrating factor of the differential equation is $\sec x$.

Question9

If A and B are arbitrary constants, then the differential equation having $y = Ae^{-x} + B\cos x$ as its general solution is

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Options:

A. $(\sin x - \cos x)\frac{d^2y}{dx^2} + 2\cos x\frac{dy}{dx} - (\sin x + \cos x)y = 0$

B. $(\cos x - \sin x)\frac{d^2y}{dx^2} + 2\cos x\frac{dy}{dx} + (\sin x + \cos x)y = 0$

C. $(\cos x + \sin x)\frac{d^2y}{dx^2} + 2\sin x\frac{dy}{dx} - (\sin x - \cos x)y = 0$

D. $(\cos x - \sin x)\frac{d^2y}{dx^2} - 2\sin x\frac{dy}{dx} + (\cos x + \sin x)y = 0$

Answer: B

Solution:

We have,

$y = Ae^{-x} + B\cos x$

On differentiating w.r.t. x, we get

$y' = -Ae^{-x} - B\sin x$

On differentiating w.r.t. x, we get

$y'' = Ae^{-x} - B\cos x$

On adding in Eqs (ii) and (iii), we get

$y'' + y' = -B(\cos x + \sin x)$

$B = \frac{-y''-y'}{\cos x+\sin x} + x$

On adding in Eqs, (i) and (iii), we get

$y'' + y = 2Ae^{-x}$

$e^{-x}A = \frac{y''+y}{2}$

$\Rightarrow y = \frac{y''+y}{2} + \frac{-y''\cos x - y'\cos x}{\cos x + \sin x}$

$\Rightarrow 2y(\cos x + \sin x) = y''\cos x + y''\sin x +$

$y\cos x + y\sin x - 2y''\cos x - 2y'\cos x$

$\Rightarrow 2y(\cos x + \sin x) = y''(\sin x - \cos x) + y$

$(\cos x + \sin x) - 2y'\cos x$

$\Rightarrow y''(\sin x - \cos x) - y(\cos x + \sin x)$

$-2y'\cos x = 0$

Replace $y'' = \frac{d^2y}{dx^2}, y' = \frac{dy}{dx}$

$\because (\sin x - \cos x)\frac{d^2y}{dx^2} - 2\cos x\frac{dy}{dx} - y$

$(\cos x + \sin x) = 0$

$\Rightarrow (\cos x - \sin x)\frac{d^2y}{dx^2} + 2\cos x\frac{dy}{dx} + y$

$(\cos x + \sin x) = 0$

Question10

The general solution of the differential equation $\frac{dy}{dx} + \frac{\sin(2x+y)}{\cos x} + 2 = 0$ is

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Options:

A. $(\sec x + \tan x)[\operatorname{cosec}(2x+y) - \cot(2x+y)] = c$

B. $\sin(2x+y)\cos x = c$

C. $\cos(2x+y)\sin x = c$

D. $(\operatorname{cosec} x-\cot x)(\sec (2 x+y)-\tan (2 x+y))=c$

Answer: A

Solution:

Given, $\frac{d y}{d x}+\frac{\sin (2 x+y)}{\cos x}+2=0$

Let $2 x+y=u$

$$2+\frac{d y}{d x}=\frac{d u}{d x} \Rightarrow \frac{d y}{d x}=\frac{d u}{d x}-2$$

$$\Rightarrow \quad \frac{d u}{d x}-2+\frac{\sin (u)}{\cos x}+2=0$$

$$\Rightarrow \quad \frac{d u}{d x}+\frac{\sin u}{\cos x}=0$$

By variable seperable

$$\Rightarrow \quad \frac{d u}{\sin u}=-\frac{d x}{\cos x}$$

$$\Rightarrow \quad \int \operatorname{cosec} u d u=-\int \sec x d x$$

$$\Rightarrow \log (\operatorname{cosec} u-\cot u)$$

$$=-\log (\sec x+\tan x)+\log C$$

$$\Rightarrow \log (\operatorname{cosec} u-\cot u)$$

$$+\log (\sec x+\tan x)=\log C$$

$$\Rightarrow(\operatorname{cosec} u-\cot u)(\sec x+\tan x)=C$$

$$\Rightarrow[\operatorname{cosec}(2 x+y)-\cot (2 x+y)]$$

$$[\sec x+\tan x]=C,$$

Question11

The general solution of the differential equation $(9 x-3 y+5) d y=(3 x-y+1) d x$ is

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Options:

A. $x-3y-\log|12x-4y+7|=C$

B. $4x-12y-\log|12x-4y+7|=C$

C. $4x-12y+\log|6x-2y+7|=C$

D. $2x-6y+\log|12x-4y+7|=C$

Answer: B

Solution:

Given,

$(9x-3y+5)dy=(3x-y+1)dx$

$$\therefore \quad \frac{dy}{dx}=\frac{3x-y+1}{9x-3y+5}$$

Let $\quad 3x-y=V$

$\Rightarrow \quad 3-\frac{dy}{dx}=\frac{dV}{dx}$

$\therefore \quad 3-\frac{dV}{dx}=\frac{V+1}{3V+5}$

$\Rightarrow \quad \frac{dV}{dx}=3-\frac{V+1}{3V+5}=\frac{8V+14}{3V+5}$

$\Rightarrow \quad \left(\frac{3V+5}{8V+14}\right)dV=dx$

$\Rightarrow \quad \frac{3}{8}\left(\frac{8V+\frac{40}{3}}{8V+14}\right)dV=dx$

$\Rightarrow \quad \frac{3}{8}\left[\frac{8V+14}{8V+14}+\frac{\frac{40}{3}-14}{8V+14}\right]dV=dx$

$\Rightarrow \quad \left[\frac{3}{8}+\left(-\frac{1}{8}\right)\left(\frac{1}{4V+7}\right)\right]dV=dx$

On integrating both sides, we get

$$\frac{3}{8}V-\frac{1}{8}\times\frac{\log|4V+7|}{4}=x+C_1$$

$\Rightarrow \quad 12(3x-y)-\log|4(3x-y)+7|$

$\Rightarrow \quad =32x+32C_1$

$\Rightarrow \quad 4x-12y-\log|12x-4y+7|=C$

Where $C=32C_1$

Question12

The general solution of the differential equation $\frac{dy}{dx} = \frac{2y^2+1}{2y^3-4xy+y}$ is

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Options:

A. $4xy^2 + 2x = y^4 + y^2 + c$

B. $2xy^2 + x = y^4 - y^2 + c$

C. $4xy^2 - 2x = y^4 + y^2 + c$

D. $4xy^2 + 2x = y^4 - y^2 + c$

Answer: A

Solution:

Given,

$$\frac{dy}{dx} = \frac{2y^2+1}{2y^3-4xy+y}$$

$$\Rightarrow \frac{dx}{dy} = \frac{2y^3-4xy+y}{2y^2+1}$$

$$\Rightarrow \frac{dx}{dy} + \left(\frac{4y}{1+2y^2}\right)x = \frac{2y^3+y}{2y^2+1}$$

This is a linear differential equation

$$\text{IF } = \exp\left(\int \frac{4y}{1+2y^2}dy\right)$$

$$= \exp\left(\log\left(1+2y^2\right)\right) = 1+2y^2$$

Now, general solution is given by

$$x \times \text{IF} = \int \left[\left(\frac{2y^3 + y}{2y^2 + 1}\right) \times \text{IF}\right] dy + C$$

$$\Rightarrow x\left(1 + 2y^2\right) = \int \left(2y^3 + y\right) dy + C$$

$$\Rightarrow x + 2xy^2 = 2\left(\frac{y^4}{4}\right) + \frac{y^2}{2} + C$$

$$\Rightarrow 4xy^2 + 2x = y^4 + y^2 + C$$

Question13

The general solution of the differential equation $\left(3x^2 - 2xy\right)dy + \left(y^2 - 2xy\right)dx = 0$ is

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Options:

A. $x^2 - xy = cy^2$

B. $y^2 - xy = cx^3$

C. $xy - x^2 = cy^3$

D. $xy - y^2 = cy^3$

Answer: C

Solution:

We have,

$$\left(3x^2 - 2xy\right)dy + \left(y^2 - 2xy\right)dx = 0$$

$$\Rightarrow \frac{dy}{dx} = -\left(\frac{y^2 - 2xy}{3x^2 - 2xy}\right) \ldots \text{(i)}$$

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

From Eq. (i), we get

$$v+x\frac{dv}{dx}=-\left(\frac{v^2x^2-2vx^2}{3x^2-2vx^2}\right)$$

$$x\frac{dv}{dx}=-\left(\frac{v^2-2v}{3-2v}\right)-v$$

$$x\frac{dv}{dx}=\frac{-v^2+2v-3v+2v^2}{3-2v}$$

$$x\frac{dv}{dx}=\frac{v^2-v}{3-2v}\Rightarrow\frac{2v-3}{v^2-v}dv=-\frac{dx}{x}$$

$$\frac{2v-1}{v^2-v}-\frac{2}{\left(v-\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}\Bigg)dv=-\frac{dx}{x}$$

$$\log\left(v^2-v\right)-\frac{2}{2\left(\frac{1}{2}\right)}\log\left(\frac{v-\frac{1}{2}-\frac{1}{2}}{v-\frac{1}{2}+\frac{1}{2}}\right)$$

$$=-\log x-\log c$$

$$\log\left(v^2-v\right)-2\log\left(\frac{v-1}{v}\right)=-(\log c+\log x)$$

$$\log\left(v^2-v\right)-\log\left(\frac{v-1}{v}\right)^2=-\log cx$$

$$\Rightarrow\log\left\{\left[v(v-1]\times\frac{v^2}{(v-1)^2}\right\}=-\log\alpha$$

$$\frac{v^3}{v-1}=\frac{1}{cx}\Rightarrow\frac{\frac{y^3}{x^3}}{\frac{y}{x}-1}=\frac{1}{cx}$$

$$\frac{y^3\cdot x}{x^3(y-x)}=\frac{1}{cx}\Rightarrow\frac{y^3}{y-x}=\frac{x}{c}$$

$$\Rightarrow\quad xy-x^2=cy^3$$

Which is the general solution of given differential equation.

Question14

The degree and order of the differential equation of the family of parabolas whose axis is the X-axis, are respectively

Options:

A. 2,2

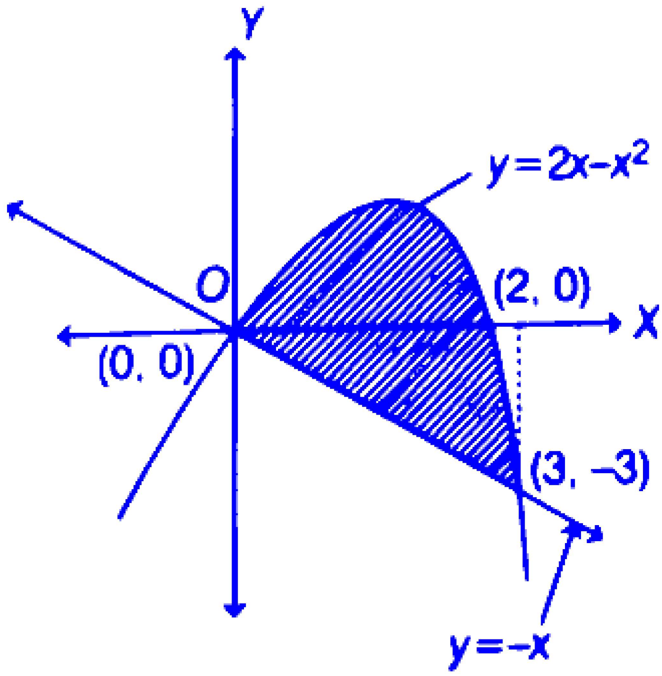
B. 2,1

C. $1, 2^{\circ}$

D. 3,2

Answer: C

Solution:



Now, equation of family of parabola whose axis is the X-axis i.e $y^2 = 4a(x + \alpha)$ where, a and α are constant.

Now, Differentiate on both sides w.r.t.

$$2y\frac{dy}{dx} = 4a \Rightarrow y\frac{dy}{dx} = 2a$$

$$\Rightarrow y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$$

Thus, order 2 and degree 1 .

Question15

The general solution of the differential equation $\left(x \sin \frac{y}{x}\right) d y=\left(y \sin \frac{y}{x}-x\right) d x$ is

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Options:

A. $\sin ^{-1}\left(\frac{y}{x}\right)=\frac{x}{2}+C$

B. $\sin \left(\frac{x}{y}\right)=\frac{x^2}{2}+C$

C. $\sin \left(\frac{y}{x}\right)=\log |x|+C$

D. $\cos \left(\frac{y}{x}\right)=\log |x|+C$

Answer: D

Solution:

Given, differential equation is

$$\left(x \sin \frac{y}{x}\right) d y=\left(y \sin \frac{y}{x}-x\right) d x$$

$$\Rightarrow \quad \frac{d y}{d x}=\frac{y \sin \frac{y}{x}-x}{x \sin \frac{y}{x}}$$

Let $\quad y=v x \Rightarrow \frac{d y}{d x}=v+x \frac{d v}{d x}$

$$\Rightarrow \quad v+x \frac{d v}{d x}=\frac{v x \sin v-x}{x \sin v}$$

$$\Rightarrow \quad v+x \frac{d v}{d x}=\frac{v \sin v-1}{\sin v}$$

$$\Rightarrow \quad x \frac{d v}{d x}=\frac{v \sin v-1}{\sin v}-v=-\frac{1}{\sin v}$$

$$\Rightarrow \quad \int \sin v d v=-\int \frac{d x}{x}-C$$

$$\Rightarrow \quad-\cos v=-\log |x|-C$$

$$\Rightarrow \quad \cos \left(\frac{y}{x}\right)=\log |x|+C$$

Question16

The general solution of the differential equation $\left(2x-10y^3\right)dy+ydx=0, y\neq 0$ is

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Options:

A. $x^2y-2y^3=C$

B. $xy^2-2y^5=C$

C. $xy^3+2y=C$

D. $xy^2+3y=C$

Answer: B

Solution:

The given differential equation is

$$\begin{aligned} &\Rightarrow \quad \left(2x-10y^3\right)dy+ydx=0 \\ &\Rightarrow \quad \tfrac{dx}{dy}=-\tfrac{2x}{y}+10y^2 \\ &\Rightarrow \quad \frac{dx}{dy}+\frac{2x}{y}=10y^2 \end{aligned}$$

Now, IF $=e^{\int\frac{2}{y}dy}=e^{2\log y}=y^2$

solution is

$$\begin{aligned} x\cdot y^2 &= \int\left(10y^2\right)\cdot\left(y^2\right)dy+C \\ &= \int 10\cdot y^4dy+C \\ &= 2y^5+C \end{aligned}$$

$\Rightarrow xy^2-2y^5=C$

Question17

If m and n are respectively the order and degree of the differential equation of the family of parabolas with origin as its focus and X-axis as its axis, then $mn-m+n=$

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Options:

A. 1

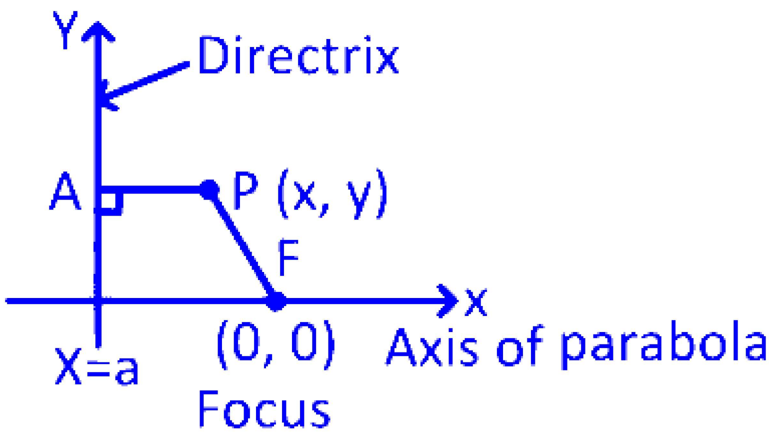
B. 2

C. 3

D. 4

Answer: C

Solution:



Let a be any arbitrary constant

$$PF = PA$$

$$\Rightarrow \quad \sqrt{x^2+y^2} = (x-a)$$

$$\Rightarrow \quad x^2+y^2 = x^2+a^2-2ax$$

$$y^2 = a^2 - 2ax$$

$$2y\frac{dy}{dx} = -2a$$

$$a = -y\frac{dy}{dx}$$

On putting the value of a (ii) in Eq. (i), we get

$$y^2 = y^2\left(\frac{dy}{dx}\right)^2 + 2xy\frac{dy}{dx}$$

$$m = 1, n = 2$$

$$mn - m + n = 3$$

Question18

The general solution of $\frac{dy}{dx}+yf'(x)-f(x)f'(x)=0$, $y\neq f(x)$ is

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Options:

A. $y=f(x)+1+ce^{-f(x)}$

B. $y=ce^{-f(x)}$

C. $y=f(x)-1+ce^{-f(x)}$

D. $y=f(x)+ce^{f(x)}$

Answer: C

Solution:

$$\frac{dy}{dx}+yf'(x)-f(x)f'(x)=0$$
$$\Rightarrow \frac{dy}{dx}+y\frac{df(x)}{dx}-f(x)\frac{df(x)}{dx}=0$$

$\Rightarrow \frac{dy}{df(x)}+y=f(x)$

Integrating factor $=e^{\int df(x)}=e^{f(x)}$

$e^{f(x)}y=\int f(x)e^{f(x)}df(x)$

$e^{f(x)}y=f(x)\int e^{f(x)}df(x)$

$-\int\left(\int e^{f(x)}dx\cdot\frac{df(x)}{df(x)}\right)df(x)$

$$e^{f(x)}y=f(x)e^{f(x)}-\int e^{f(x)}df(x)+c$$

$e^{f(x)}y=f(x)e^{f(x)}-e^{f(x)}+c$

$y=f(x)-1+ce^{-f(x)}$
